Reduction of Solid Rocket Data when Pressure-Time History Is Nonneutral

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Nomenclature

A_e = nozzle exit area
A_t = nozzle throat area
C^* = characteristic velocity
g = gravitational acceleration
I = delivered impulse
I_{sp} = specific impulse
$I_{sp,d}$ = delivered specific impulse
m = propellant mass
$\dot{m}$ = nozzle flow rate
p = pressure
$\bar{p}_m$ = mass averaged pressure
$\bar{p}_t$ = time averaged pressure
$\bar{p}_r$ = mean pressure for burning rate
$\bar{p}$ = mean pressure
r = burning rate
t = time
t_a = action time
t_b = burn time
t_I = time when p first becomes 0.05 $\bar{p}_t$
w = propellant web
$(\bar{\ })$ = denotes a mean
ϵ = nozzle expansion ratio

REDUCING pressure- and thrust-time, propellant mass and web, and nozzle geometry data to determine mean burning rate, mean characteristic velocity, delivered (mean) specific impulse, mean pressure, and mean nozzle geometry is reasonably straightforward with neutral pressure-time histories and minimal nozzle erosion because pressure and nozzle geometry are unique in this situation. However, when the pressure-time history is nonneutral, pressure (and usually nozzle geometry) is no longer unique, and choosing the correct mean pressure becomes a problem. It is important to note that functional relationships exist between dependent (r, C^*, I_{sp}) and independent (p, ϵ, etc.) variables. Indeed, one purpose of performance testing is to determine these relationships experimentally. Consequently, when means of dependent variables are defined, means of the independent variables must be derived from these definitions. In other words, when the independent variables (pressure, etc.) vary during a test firing, their means cannot be arbitrarily defined. In Ref. 1, constraints are placed upon independent variable variations during a test. If variations exceed constraints, these data are unacceptable. Although this approach eliminates nonneutral independent variable problems and yields quality performance data, it is not cost effective. Bought and paid for information is often unacceptable.

The objectives of this work are twofold. First, to derive general equations defining consistent independent variable means when motor operation is in the quasi-steady regime and pressure and throat area are the independent variables. Second, to obtain first approximation "solutions" to these equations.

Reference 1 defines mean burning rate, mean characteristic velocity,† and delivered specific impulse as

$$\bar{r} = w/t_b \tag{1}$$

$$\bar{C}^* = g\int_{t_I}^{t_I+t_a} A_t p \ \ dt/m_p \tag{2}$$

$$I_{sp,d} = \int_{t_I}^{t_I+t_a} F \ \ dt/m_p \tag{3}$$

The mean independent variables are defined as‡

$$\bar{p}_t = \int_{t_I}^{t_I+t_a} p \ \ dt/t_a \tag{4}$$

$$\bar{A}_t = [A_t(t_I+t_a)+A_t(0)] \ /2 \tag{5}$$

$$\bar{\epsilon} = A_e(t_I+t_a)/\bar{A}_t \tag{6}$$

Since functionals r (p), C^* (p), and $I_{sp}(p, \epsilon)$ are presumed to exist, consistent means for p and ϵ require that

$$\bar{r} = r(\bar{p}_r) \tag{7a}$$

$$\bar{C}^* = C^* \ (\bar{p}_C) \tag{7b}$$

$$I_{sp,\,d} = I_{sp}(\bar{p}_I, \bar{\epsilon}_I) \tag{7c}$$

where $\bar{p}_r$, $\bar{p}_C$, and $\bar{p}_I$ are mean pressures for burning rate, characteristic velocity, and specific impulse, respectively, and $\bar{\epsilon}_I$ is the mean expansion ratio for specific impulses. Since

$$w = \int_{t_I}^{t_I+t_b} r(p) \ dt$$

Eq. (1) can be rewritten as

$$r(\bar{p}_r) = \int_{t_I}^{t_I+t_b} r(p) \ dt/t_b \tag{8}$$

For quasi-steady conditions

$$I = \int_{t_I}^{t_I+t_a} I_{sp} dm, \S \qquad m = \int_{t_I}^{t_I+t_a} \dot{m} \ dt$$

and $\dot{m} = A_t p \ g/C^*$. Therefore, Eqs. (2) and (3) can be rewritten as

$$C^*(\bar{p}_C) = \int_{t_I}^{t_I+t_a} A_t p \ dt \ / \int_{t_I}^{t_I+t_a} [A_t p/C^* \ (p)] \ dt \tag{9}$$

†Equation (2) is a generalization of the more commonly employed

$$\bar{C}^* = g\bar{A}_t \int_{t_I}^{t_I+t_a} p \ \ dt/m$$

‡For nonneutral pressure-time histories, Ref. 1 suggests that the mass-averaged pressure

$$\bar{p}_{ms} \simeq \int_{t_I}^{t_I+t_a} p^2 \ dt / \int_{t_I}^{t_I+t_a} p \ dt$$

replace the time-averaged pressure $\bar{p}_t$.

§This shows that specific impulse is a mass averaged quantity. Consequently, it has been argued that $\bar{p}_m$ should be employed with $I_{sp,d}{}^I$, i.e., $\bar{p}_I = \bar{p}_m$. Following this same argument, $\bar{p}_r = \bar{p}_t$.

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$$I_{sp}(\bar{p}_I,\bar{\epsilon}_I)=g\int_{t_I}^{t_I+t_a}[I_{sp}(p,\epsilon)A_t p/C^*(p)]\,dt/m \tag{10}$$

Equation (10) does not specify $\bar{p}_I$ and $\bar{\epsilon}_I$ uniquely. Therefore, it is necessary to introduce an arbitrary definition. Since specific impulse and characteristic velocity are both thermodynamic parameters, it is logical that both should possess the same mean pressure. Consequently assume

$$\bar{p}=\bar{p}_I=\bar{p}_C \tag{11}$$

Examination of Eqs. (8-10) shows the following: a) they are implicit equations; b) the function desired must be known to determine the desired mean; c) since r, C^*, and I_{sp} are, in general, neither proportional to p nor independent of p, $\bar{p}_r\neq\bar{p}\neq\bar{p}_m\neq\bar{p}_t$ for nonneutral pressure-time histories; and d) when the pressure-time history is neutral, $\bar{p}_r=\bar{p}=\bar{p}_m=\bar{p}_t$. Item b) shows that when the pressure-time history is nonneutral motor test data cannot be reduced on a single firing basis. If the dependent variable is assumed to be an M parameter function of the independent variables, at least M tests are required, and data for the M firings must be reduced simultaneously. Items c) and d) show that although the CPIA recommendations are exact for neutral pressure-time histories, they are not precisely valid for nonneutral pressure-time histories.

First approximation "solutions" to Eqs. (8-10) are obtained by assuming

$$r=ap^n,\quad \bar{C}^*/C^*=1+\alpha(p-\bar{p})\ \square$$

$$I_{sp}/I_{sp,d}=1+\beta(p-\bar{p})+\delta(\epsilon-\bar{\epsilon}).$$

Then, Eqs. (8-10) become¶

$$\bar{p}_r=\left[\int_{t_I}^{t_I+t_b}p^n\,dt/t_b\right]^{1/n} \tag{12}$$

$$\bar{p}=\int_{t_I}^{t_I+t_a}A_t p^2\,dt\ \Big/\int_{t_I}^{t_I+t_a}A_t p\,dt \tag{13}$$

$$\bar{\epsilon}=\beta(\bar{p}_m-\bar{p})/\delta+A_e\Big[\bar{p}_t(1+\alpha\bar{p})+\alpha\int_{t_I}^{t_I+t_a}p^2\,dt\Big]\Big/\int_{t_I}^{t_I+t_a}A_t p\,dt \tag{14}$$

If A_t is constant, $\bar{p}=\bar{p}_m$ and $\bar{\epsilon}=A_e/A_t$. Therefore, the CPIA recommendations for nonneutral pressure-time histories are valid as first approximations.

¶Equation (12) has been obtained previously by Brock.[2]

The magnitude of the errors involved with employing $\bar{p}_t$ for $\bar{p}_r$ [Eq. (12)] and $\bar{p}$ [Eq. (13)] can be readily estimated. Expanding the integrands of Eqs. (12) and (13) in a Taylor's series about $p=\bar{p}_t$ gives

$$p^n=p_t^{-n}\Big[1+\sum_{k=1}^{\infty}\prod_{j=0}^{k-1}(n-j)\frac{(\Delta p/\bar{p}_t)^k}{k!}\Big] \tag{15}$$

When $n<1$ this series has alternating sign. Consequently, when convergent,

$$p^n\simeq 1+n(\Delta p/\bar{p}_t)+n(n-1)(\Delta p/\bar{p}_t)^2/2! \tag{16}$$

with error less than $|n(n-1)(n-2)(\Delta p/\bar{p}_t)^3/3!^3|$. Substitution of Eq. (16) into Eq. (12), integrating, and applying the mean value theorem for integrals gives

$$\bar{p}_r\simeq\bar{p}_t\left[1+n(n-1)\ [\overline{(\Delta p/\bar{p}_t)^2}]\ /2\right]^{1/n} \tag{17}$$

Substitution of Eq. (16) with $n=2$ into Eq. (13), assuming A_t is constant, integrating, and applying the mean value theorem for integrals gives

$$\bar{p}=\bar{p}_t\{\ 1+[\ \overline{(\Delta p/\bar{p}_t)^2}\ ]\ \} \tag{18}$$

Examination of Eqs. (17) and (18) shows that when $\Delta p/\bar{p}_t$ is small, $\bar{p}_r$ and $\bar{p}$ agree with $\bar{p}_t$ to first-order accuracy. However, when $\Delta p/\bar{p}_t$ is large, significant deviations can occur. Calculations presented by Brock[2] support this.

In summary, this work has shown that consistant reduction of motor test data when the pressure-time history is nonneutral falls outside conventional procedure. As long as deviations from neutrality are small, errors relative to established procedure are small. However, when deviations become large, significant errors can result. This work has assumed that motor operation falls in the quasi-steady regime and that the pressure and throat area are the only independent variables. However, burning rate depends on flow over the burning surface and the thermodynamic parameters depend at least upon stay time and nozzle geometry. Therefore, what has been presented here is just a ripple on the surface of the basic problem of extracting all possible truth from available data.

References

[1] "Recommended Procedure for the Measurement of Specific Impulse of Solid Propellants," CPIA Publication 174, Aug. 1968, Chemical Propulsion Information Agency, Silver Spring, Md.

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[3] Taylor, A.E., *Advanced Calculus*, Ginn and Company, Boston, Mass., 1955, p. 563.